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QUASI RENEWAL PROCESSES FOR SOFTWARE AND HARDWARE SYSTEMS WITH COMMON CAUSE FAILURE

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ABSTRACT

A software/hardware system consisting of one software component and N hardware components which are subject to individual and common cause failures is analyzed. The renewal theory has been applied to determine renewals that bring a component back into the system as-good-as new one. In this paper, we investigate quasi-renewal processes of repairs and replacements for developing the warranty cost models. The reliability and other measures are obtained for performance evaluation of the system. Numerical example is given to illustrate the computational tractability of the model with the help of MATLAB software.

Keywords: Software/hardware system, Warranty cost, K-out-of-N system, Common cause failure, Quasi-renewal processes.

1. INTRODUCTION

A warranty is an pledge from a manufacturer to a consumer that the product sold is agreement to behave convincingly upto a specified period of time i.e. the warranty period. In case when an item does not perform satisfactorily as specified, the dealer/producer is responsible to repair or replace it by a new one. The main object of a warranty is to provide protection to both manufacturers as well as buyer. The term of warranty is not only the concern of the consumers but also increases the sales and reputation of the manufacturers. Now-a-days with the competitive global market, the warranty has become more powerful aspect to boost the sales and revenue. Many different nature of warranty policies such as free replacement repair policy, pro-rata warranty policy, etc. exist in the literature. In free replacement repair policy, the manufacturer guarantees to repair or provides replacement for aborted items free of cost upto specified warranty period from the date of initial purchase. In pro-rata warranty policy, the producer admits to compensate a fraction of the investment of the items which fail from the specified time of the basic earn. Many researchers have studied the warranty cost models in different frameworks. Ascher and Feingold (1984), Abdel-Hameed (1995) and Hunter (1996) gave the mathematical techniques for warranty analysis. Pham (2003) analyzed the software reliability and cost models. Rahman and Chattopadhyay (2006) gave the review of long term warranty policies. Yun et al. (2008) studied warranty servicing with imperfect repair. Wu et al. (2009) considered optimal price, warranty length and production rate for free replacement policy in the static demand market. Yang et al. (2009) considered cost-oriented task allocation and hardware redundancy policies in heterogeneous distributed computing systems considering software reliability. Srinivas et al. (2009) analyzed the influence of delivery times on repairable k-out-of-N systems with spares. Zhu et al. (2010) proposed the availability optimization of the system subject to competing risk. Mo at al. (2015) and Eryilmaz (2018) discussed the optimal model considering the system having multiple types of components also having nonidentical failure time distribution. Zhao et al. (2018) suggested a optimized model with finite warranty and imperfect repairs according to the field requirements of the product. Zhang et al. (2018) analyzed stochastic dependency of two component based system and discussed the warranty cost calculation from the producer and buyer's aspect. Sheng and Ke (2020) investigated the generalized uncertain k-out-of-n systems. He also considered that components might be in multiple states.

In reliability literature, renewal theory provides many variants of replacement/repair policies for maintainability of the system. Shaked and Zhu (1992) gave some results on the block replacement policies using renewal theory. Blischke and Murthy (1994), Wang and Pham (1996) and Murthy et al. (2004) discussed the quasi-renewal process and its applications in imperfect maintenance. Pham and Wang (2001) proposed a quasi-renewal process for the software reliability and testing costs. Yanez et al. (2002) discussed generalized renewal process for the analysis of repairable systems with limited failure experience. Park and Pham (2008) developed the warranty system-cost model using quasi-renewal processes. Samatli-Pac and Taner (2009) discussed the role of repair strategy in warranty cost minimization via quasi-renewal processes. Samatlı-Paç et al. (2009) and Park and Pham (2010) studied altered quasi-renewal concepts for modeling renewable warranty costs with imperfect repairs. In 2015, Jung et al. attracted the attention by elaborating the extended renewing warranty policies which is to be applied when original two phase warranty expires. Park and Pham (2016) discussed the maintenance cost under the renewable and non-renewable warranty policies. Recently, Park et al. (2020) described the warranty policies considering that user will receive full refund if the failure cannot be get repaired during the warranty period

The warranty cost depends on the terms of the warranty and is calculated by the manufacturer as per servicing a claim under warranty. In this paper, we suggest a free replacement warranty policy according to which if an item fails, it is replaced by a new without paying any cost (i.e. free of charge) because the item is non repairable. The replacement occurs according to a renewal process. The number of failures during the warranty period is mathematically calculated based on quasi renewal process. We evaluate the representative cost functions to evaluate the efficacy of policies. The rest of the paper is structured as follows. Section 2 deals with model description by stating the requisite assumptions and nomenclatures. Section 3 provides the mathematical analysis of the model. In section 4, we describe the warranty policy with repair. In section 5, numerical results are provided. Finally in section 6, the conclusion is drawn.

2. MODEL DESCRIPTION

We consider the quasi-renewal analysis of the distribution function of the number of product failures of a multi-component repairable system within a warranty period w. A replacement service would be available during the warranty period by introducing two quasi renewal concepts (i) altered quasi renewal process and (ii) mixed quasi renewal process. Appling quasi renewal process, the cost analysis is performed for K-out-of-N system consisting of N hardware components and one software component. The system components may fail individually or due to common cause failure. For modeling purpose, the following assumptions are being made.

Assumptions

- The repair and replacement are different aspects.
- The repair cost and replacement cost are stable. Also, repair and replacement service time are negligible.
- All warranty claims are accomplished and are authentic.
- The repairs are immature and the repair process is modeled by a quasi-renewal process.
- The isolation time is considered as negligible.
- The failure product may be replaced by new one during warranty period.

Nomenclatures

w	:	Warranty period.
T	:	R.V. denoting time.
λ_C	:	Common cause failure.
β	:	Parameter for QRP.
N_{system}	:	System failures.
$R_C(w)$	:	The inter-failure time function of common cause.
$N_h(t)$,$N_s(t)$, $N_c(t)$	:	The number of hardware failures, software failures and failure due to common cause in (0,t], respectively.
$f_h(.)$, $F_h(.)$, $R_h(.)$	:	pdf, cdf and reliability function of hardware failure time within a warranty period w, respectively.
$f_s(.)$, $F_s(.)$, $R_S(.)$	:	pdf, cdf and reliability function of software failure time within a warranty period w, respectively.
$f_{ih}(.)$, $F_{ih}(.)$, $R_{ih}(.)$	:	pdf, cdf and reliability function of the hardware component failure times after $(i-1)^{th}$ repair/replacement within a warranty period w, respectively.

$f_{is}(.)$, $F_{is}(.)$, $R_{is}(.)$	:	pdf, cdf and reliability function of the software component failure times after $(i-1)^{th}$ repair/replacement within a warranty period w, respectively.
C(w)	:	Warranty cost of the system for a warranty period w.
c_h , c_s, c_0	:	Warranty cost for repairs/replacements within a warranty period w, for hardware, software and common cause failure, respectively.

3. THE ANALYSIS

Following Park and Pham (2008) pmf of N_h and N_s are given by

$$P[N_h = n_h] = \left(\prod_{i=1}^{n_h} F_{ih}(w)\right)\left(R_{(n_h+1)h}(w)\right) \tag{1}$$

and

$$P[N_s = n_s] = \left(\prod_{j=1}^{n_s} F_{js}(w)\right)\left(R_{(n_s+1)s}(w)\right) \tag{2}$$

In this section we evaluate the expected number of system failure due to hardware, software components and common cause failures. Under the imperfect repair, the reliability functions for hardware a component and software respectively, are obtained as

$$R_h(w) = \prod_{i=1}^{n_h}\left(R_{ih}(w)\right) = \prod_{i=1}^{n_h}\left(1 - \int_0^x \beta_{ih} f(\beta_{ih} x)dx\right) \tag{3}$$

$$R_s(w) = \prod_{j=1}^{n_s}\left(R_{js}(w)\right) = \prod_{j=1}^{n_s}\left(1 - \int_0^x \beta_{js} f(\beta_{js} x)dx\right) \tag{4}$$

We also define

$$R_C(w) = 1 - F_C(w) = 1 - \int_0^x \lambda_C f(\lambda_C x)dx \tag{5}$$

3.1 Series System

The reliability function of the system when N hardware components arranged in series is given by

$$R(w) = \left(\prod_{i=1}^{n_h} R_{ih}(w)\right)^N \left(\prod_{j=1}^{n_s} R_{js}(w)\right) R_c(w)$$

$$= \left[\prod_{i=1}^{n_h}\left(1 - \int_0^x \beta_{ih} f(\beta_{ih} x)dx\right)\right]^N \left[\prod_{j=1}^{n_s}\left(1 - \int_0^x \beta_{js} f(\beta_{js} x)dx\right)\right]\left(1 - \int_0^x \lambda_c f(\lambda_c x)dx\right) \tag{6}$$

3.2 K-out-of-N System

When the hardware components are arranged in a K-out-of-N system along with one software component, the reliability function is obtained as the probability of having at least K functioning hardware units out of N and software in functioning state along with no failure due to common cause. Thus, we obtain

$$R(w) = \sum_{k=K}^{N} \binom{N}{k} [R_h(w)]^k [1 - R_h(w)]^{N-k} . R_s(w). R_c(w)$$

$$= \sum_{k=K}^{N}\binom{N}{k}\left[\prod_{i=1}^{n_h}\left(1-\int_0^x \beta_{ih} f(\beta_{ih}x)dx\right)\right]^k \left[1-\prod_{i=1}^{n_h}\left(1-\int_0^x \beta_{ih} f(\beta_{ih}x)dx\right)\right]^{N-k} \prod_{j=1}^{n_s}\left(1-\int_0^x \beta_{js} f(\beta_{js}x)dx\right)\left(1-\int_0^x \lambda_c f(\lambda_c x)dx\right) \tag{7}$$

The probability that the system is not working is given by

$$\begin{aligned} P(w) &= \text{Prob.}(\text{system is not working}) \\ &= \sum_{k=0}^{K-1}\binom{N}{k}\{R_h(w)\}^k\left(\overline{R_h(w)}\right)^{N-k}.R_s(w).R_c(w) \\ &+ \sum_{k=K}^{N}\binom{N}{k}\{R_h(w)\}^k\left(\overline{R_h(w)}\right)^{N-k}\left[\overline{R_s(w)}.R_c(w)+R_s(w).\overline{R_c(w)}\right] \end{aligned} \tag{8}$$

where $\overline{R_s(w)} = 1 - R_s(w)$ and $\overline{R_C(w)} = 1 - R_C(w)$

Thus

$$\begin{aligned} P(w) = &\sum_{k=0}^{K-1}\binom{N}{k}\left[\prod_{i=1}^{n_h}\left(1-\int_0^x \beta_{ih} f(\beta_{ih}x)dx\right)\right]^k \left[1-\prod_{i=1}^{n_h}\left(1-\int_0^x \beta_{ih} f(\beta_{ih}x)dx\right)\right]^{N-k} \prod_{j=1}^{n_s}\left(1-\int_0^x \beta_{js} f(\beta_{js}x)dx\right) \\ &.\left(1-\int_0^x \lambda_c f(\lambda_c x)dx\right) + \sum_{k=K}^{N}\binom{N}{k}\left[\prod_{i=1}^{n_h}\left(1-\int_0^x \beta_{ih} f(\beta_{ih}x)dx\right)\right]^k \left[1-\prod_{i=1}^{n_h}\left(1-\int_0^x \beta_{ih} f(\beta_{ih}x)dx\right)\right]^{N-k} \\ &.\left[\left\{1-\left(\prod_{j=1}^{n_s}\left(1-\int_0^x \beta_{js} f(\beta_{js}x)dx\right)\right)\right\}.\left(1-\int_0^x \lambda_c f(\lambda_c x)dx\right)\right. \\ &\left.+\left\{\prod_{j=1}^{n_s}\left(1-\int_0^x \beta_{js} f(\beta_{js}x)dx\right)\right\}\left\{\int_0^x \lambda_c f(\lambda_c x)dx\right\}\right] \end{aligned}$$

Hence, the expected number of system failures due to hardware component failure is given by

$$E(N_h) = \sum_{n_h=1}^{\infty}\left[n_h.\sum_{k=0}^{K-1}\binom{N}{k}(R_h(w))^k(1-R_h(w))^{N-k}.R_s(w).R_c(w)\right] \tag{9}$$

Expected number of system failures due to software failure is given by

$$E(N_s) = \sum_{n_s=1}^{\infty}\left[n_s.\sum_{k=K}^{N}\binom{N}{k}(R_h(w))^k\left(\overline{R_h(w)}\right)^{N-k}.\overline{R_s(w)}.R_c(w)\right] \tag{10}$$

Expected number of system failures due to common cause failure is given by

$$E(N_C) = \sum_{n_C=1}^{\infty}\left[n_C.\sum_{k=K}^{N}\binom{N}{k}(R_h(w))^k\left(\overline{R_h(w)}\right)^{N-k}.R_s(w).\overline{R_c(w)}\right] \tag{11}$$

Expected number of system failures is given by

$$E(N_{system}) = E(N_h) + E(N_s) + E(N_C) \tag{12}$$

Now we derive the variance of the number of system failures due to hardware component failures as follows:

The second moment of the number of system failures due to hardware failure is

$$E(N^2{}_h) = \sum_{n_h=1}^{\infty}\left[n_h^2 . \sum_{k=0}^{K-1}\binom{N}{k}(R_h(w))^k (1-R_h(w))^{N-K} . R_s(w) . R_c(w)\right] \tag{13}$$

Therefore, the variance of the number of system failures due to failure of hardware component is given by

$$\text{Var}\,(N_h) = E(N_h^2) - [E(N_h)]^2 \tag{14}$$

Similarly, we can obtain the variance of the number of system failures due to software component failure as

$$\text{Var}\,(N_S) = E(N_S^2) - [E(N_S)]^2 \tag{15}$$

where

$$E(N^2{}_s) = \sum_{n_s=1}^{\infty}\left[n^2{}_s . \sum_{k=K}^{N}\binom{N}{k}(R_h(w))^k \overline{(R_h(w))}^{N-k} . \overline{R_s(w)} . R_c(w)\right] \tag{16}$$

Also

$$\text{Var}\,(N_C) = E(N_C^2) - [E(N_C)]^2 \tag{17}$$

The variance of the number of repair services for the system is given by

$$\text{Var}\,(N_{system}) = E(N_{system}^2) - [E(N_{system})]^2 \tag{18}$$

Let c_h , c_S and c_0 be the repair cost per failure due to hardware failure, software failure and common cause failure, respectively. The expected warranty cost is given by

$$E(C(w)) = c_h . E(N_h) + c_S . E(N_S) + c_o . E(N_C) \tag{19}$$

The variance of warranty system cost for software can be obtained as

$$\text{Var}(C(w)) = c_h^2 . \text{Var}(N_h) + c_S^2 . \text{Var}(N_S) + c_o^2 . \text{Var}(N_C) \tag{20}$$

4. WARRANTY POLICY WITH REPAIR

In this section we consider the warranty repair service and do not take into consideration the warranty replacement service. For illustration purpose we consider a dissimilar hardware system having 3-out-of-4 configuration of hardware units along with one software unit. Let x_1, x_2, x_3, x_4 be the indicators denoting that the hardware components 1 through 4 are working respectively. Also x_S and x_C indicate that there is no failure due to software and common cause, respectively. $\overline{x_i}$ denotes the complement events so that $\overline{x_i} = 1 - x_i$.

Then the reliability of the system is given by

$$R = \left[P(x_1\, x_2\, x_3 \overline{x_4}) + P(x_1\, x_2\, \overline{x_3} x_4) + P(x_1\, \overline{x_2} x_3\, x_4) + P(\overline{x_1} x_2\, x_3\, x_4) + P(x_1\, x_2\, x_3 x_4)\right] P(x_S) P(x_C)$$

(21)

$$R = [P(N_1 = n_1)P(N_2 = n_2)P(N_3 = n_3)\overline{P(N_4 = n_4)} + P(N_1 = n_1)P(N_2 = n_2)\overline{P(N_3 = n_3)}P(N_4 = n_4) + P(N_1 = n_1)\overline{P(N_2 = n_2)}P(N_3 = n_3)P(N_4 = n_4) + \overline{P(N_1 = n_1)}P(N_2 = n_2)P(N_3 = n_3)P(N_4 = n_4) + P(N_1 = n_1)P(N_2 = n_2)P(N_3 = n_3)P(N_4 = n_4)]P(N_S = n_S).P(N_C = n_C)$$

(22)

where

$$P[N_h = n_h] = \left[\left(\prod_{i=1}^{n_h}\left(\int_0^x \beta_{ih} f(\beta_{ih} x)dx\right)\right)\left(1 - \int_0^x \beta_{(n_h+1)h} f\left(\beta_{(n_h+1)h}.x\right)dx\right)\right]P(x_S)P(x_C)$$

$$, h = 1,2,3,4$$

Also $P(x_S)P(x_C) = R_S(w)R_C(w)$

$$= \prod_{j=1}^{n_s}\left(1 - \int_0^x \beta_{js} f(\beta_{js} x)dx\right)\left(1 - \int_0^x \lambda_C f(\lambda_C x)dx\right)$$

The expected warranty cost is given by:

$$E(C) = cE(N)$$

where $c = c_h + c_s + c_0$

4(i). K-R-out-of-N System

Let us consider K-R-out-of-N system. The probability that a system is working is given by

$$P_{(\text{system working})} = \sum_{l=K}^{R}\binom{N}{l}(R_h(w))^l(1 - R_h(w))^{N-l}.R_s(w).R_c(w) \tag{23}$$

Now the probability that a system is not working is given by

$$P_{(\text{system not working})} = \sum_{l=0}^{K-1}\binom{N}{l}(R_h(w))^l(1 - R_h(w))^{N-l}.R_s(w).R_c(w) + \sum_{l=R+1}^{N}\binom{N}{l}(R_h(w))^l(1 - R_h(w))^{N-l}.R_s(w).R_c(w) + \sum_{l=K}^{R}\binom{N}{l}(R_h(w))^l\left(\overline{R_h(w)}\right)^{N-l}\left[\overline{R_s(w)}.R_c(w) + R_s(w).\overline{R_c(w)}\right] \tag{24}$$

The expected number of system failure due to hardware component failure is given by

$$E(N'_h) = \sum_{n_h=1}^{\infty}\left[n_h\left\{\sum_{l=1}^{K-1}\binom{N}{l}(R_h(w))^l(1 - R_h(w))^{N-l}.R_s(w).R_c(w) + \sum_{l=R+1}^{N}\binom{N}{l}(R_h(w))^l(1 - R_h(w))^{N-l}.R_s(w).R_c(w)\right\}\right] \tag{25}$$

So, the expected number of system failure due to software component failure is given by

$$E(N'_s) = \sum_{n_s=1}^{\infty}\left[n_s \left(\sum_{l=K}^{R} \binom{N}{l} (R_h(w))^l \overline{(R_h(w))}^{N-l} . \left(\overline{R_s(w)}.R_c(w)\right)\right)\right] \tag{26}$$

The expected number of system failure due to common cause is given by

$$E(N'_C) = \sum_{n_C=1}^{\infty}\left[n_C \left(\sum_{l=K}^{R} \binom{N}{l} (R_h(w))^l \overline{(R_h(w))}^{N-l} . \left(\overline{R_s(w)}.R_c(w)\right)\right)\right] \tag{27}$$

Expected number of system failure is given by

$$E(N) = E(N'_h) + E(N'_s) + E(N'_C) \tag{28}$$

Now with the help of above results we can calculate the variance of the number of system failure, the variance of the number of repair services, the expected warranty cost and the variance of warranty system cost, respectively as obtained for K-out-of-N system.

5. NUMERICAL RESULT

In this section, we provide numerical results by coding computer program in 'MATLAB' software to examine the validity and tractability of analytical results of the proposed model by taking an illustration. We consider a 3-out-of-4 system and compute numerical results for total expected cost, standard deviation and coefficient of variance of the system. The life time of the components follows exponential distribution. Warranty cost and other parameters are taken as β_{1h}=0.7, β_{2h}=0.3, β_{3h}=0.4, β_{4h}=0.2, β_{6h}=0.1, β_{8h}=0.2, β_{9h}=0.8, β_{12h}=0.3, β_{6h}=0.4, β_{20h}=0.2, β_s=0.8, λ_c=.9, c=1500.

For different values of β_{1h}, $\beta_{s,}$ λ_c, the total expected cost is shown in figs 1(i)-(iii). Figs 2(i)-(iii) and 3(i)-(iii) show the standard deviation and coefficient of variance respectively with respect to the warranty period. From figs (i)-(iii), we can see that total expected cost decreases with warranty period. From figs 1(ii) and 1(iii), it is clear that as β_s and λ_c increase, the expected cost decreases.

The standard deviation graphs for different parameters with respect to warranty period are displayed in figs 2(i)-(iii). The value of standard deviation initially increases sharply and after some time it decreases slightly. In these figs, there is no significant effect of increasing the values of β_{1h} and β_s, however in fig. 2(iii), on increasing the λ_c, initially there is no change but after some time standard deviation decreases.

In figs 3(i)-(iii), we exhibit the coefficient of variance (CV) with respect to the warranty period. The value of CV initially increases sharply and after some time it increases slowly. It is also seen that CV remains almost constant as β_{1h} increases. But as β_s and λ_c increase, the value of CV reveals the increasing trend with respect to the warranty period.

6. CONCLUSION

Warranty provides an assurance to the customers regarding product quality and warranty policies. Warranty claims that if any repairable product fails during the warranty period then manufacturer has the choice to repair or replace that with the new one. The product's cost depends on various factors such as warranty policies, testing strategies and efforts, quality ans reliability of the product and time used by the manufacturer. There is huge scope for future research in this area. In this paper warranty cost models have been developed at a component level by using renewal, replacement and repair. The quasi-renewal processes used provides more realistic results for warranty cost model of K-out-of-N systems. The discussed results in this article can be used in practice for designing optimal warranty policies and would be helpful for vendors and buyers.

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